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Scattering of photons by plasmas

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Abstract. Thermodynamic Green function techniques are applied to the Compton scatter of photons by electrons in a hot plasma. An expression is obtained for the scattering rate, including relativistic effects but neglecting coupling to ions or other electrons; this restricts the formula to high-energy photons and non-zero scattering angles. The method of calculating explicit cross sections is illustrated by deducing the known result for Doppler-broadened scattering from free electrons in the non-relativistic case.

1. Introduction

The theory of incoherent light scattering by density fluctuations in a plasma has been approached in a variety of ways (Bernstein *et al.* 1964, Bekefi 1966). One of the most fundamental approaches to this subject is that of Dubois and his co-workers (Dubois *et al.* 1963, Dubois and Gilinsky 1964 a, b) which makes use of modern Green function and diagram techniques to obtain results from quantum field theory. The papers referred to are based on a non-relativistic formalism which the authors use on grounds of simplicity (Dubois *et al.* 1963, Dubois 1967). However, it seems to the present author that the computational difficulties of the fully relativistic theory are more than outweighed by its generality and close correspondence to the usual formulation of quantum electrodynamics. The calculation presented here is shallower than that of Dubois and Gilinsky (1964 a, b) in that it considers scattering from electrons and ignores coupling to ions. It is thus restricted to high-frequency photon energy loss or large-angle scattering, although within simple and explicit approximations the method allows any plasma temperature and any high energy of incident photon.

2. Formulation of the scattering rate

In the usual formulation of quantum electrodynamics (Mandl 1959) the Compton

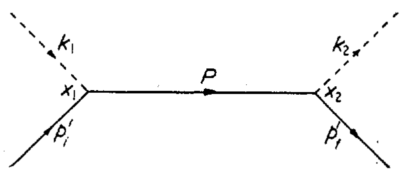

Figure 1. Compton direct scatter.

direct scatter amplitude corresponding to figure 1 is

$$M_1 = e^2 \iint \mathrm{d}x_1{}^4\, \mathrm{d}x_2{}^4 \langle p_f', k_2 | \bar{\psi}(x_2) A\!\!\!/(x_2) \underbrace{\psi(x_2)\bar{\psi}}(x_1) A\!\!\!/(x_1)\psi(x_1) | p_i', k_1 \rangle$$

$$= -\frac{\mathrm{i}e^2}{(2\pi)^4} \sum_l \int \mathrm{d}x_1{}^4\, \mathrm{d}x_2{}^4\, \mathrm{d}P^4 (4V^2\omega_1\omega_2)^{-1/2} \langle f|\bar{\psi}_\alpha(x_2)|l\rangle \{\epsilon\!\!/^{(n)}(\mathrm{i}P\!\!\!/ + m)^{-1}\epsilon\!\!/^{(m)}\}_{\alpha\beta}$$

$$\times \langle l|\psi_\beta(x_1)|i\rangle \exp[\mathrm{i}\{k_1x_1 - k_2x_2 + P(x_2 - x_1)\}] \qquad (1)$$

where $|i\rangle$, $|f\rangle$ are the initial and final states of the many-electron system and l is summed over a complete set of such states. ω_i is the energy component of the four-vector k_i, ϵ_μ the photon polarization and V the volume of the system.

If we put $p_i = (\boldsymbol{p}_i', E_i'-\mu)$, μ being the plasma chemical potential, a homogeneous system requires

$$\langle l|\psi(x)|i\rangle = \langle l|\psi|i\rangle \exp(\mathrm{i}p_i x) \tag{2}$$

with $\psi \equiv \psi(0)$ and similarly for $\bar{\psi}$.

Using (1) and (2) we can obtain

$$M_1 = \sum_l -\frac{\mathrm{i}e^2(2\pi)^4}{2V(\omega_1\omega_2)^{1/2}}\langle f|\bar{\psi}_\alpha|l\rangle {}_1\Gamma_{\alpha\beta}\langle l|\psi_\beta|i\rangle\delta^4(k_2-k_1+p_f-p_i)$$

where

$${}_1\Gamma_{\alpha\beta} = [\not{e}^{(n)}\{\mathrm{i}(\not{p}_i+\not{k}_1)+m\}^{-1}\not{e}^{(m)}]_{\alpha\beta}. \tag{3}$$

There is an exchange diagram, shown in figure 2, which for Compton scatter is comparable in magnitude with M_1. Interchange of the photon operators leads to a contribution M_2

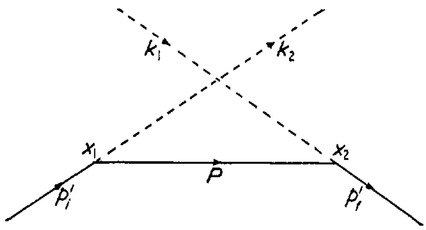

Figure 2. Compton exchange scatter.

which is equal to M_1 with ${}_1\Gamma$ replaced by

$${}_2\Gamma = \not{e}^{(m)}\{\mathrm{i}(\not{p}_i-\not{k}_2)+m\}^{-1}\not{e}^{(n)}. \tag{4}$$

We define $\tilde{\Gamma} = \gamma_4\Gamma^\dagger\gamma_4$ with $\Gamma^\dagger$ the Hermitian conjugate of Γ.

The scattering rate requires the square of the total amplitude $M = M_1+M_2$ which introduces a squared δ-function into the expression. This is interpreted in the usual way in terms of the volume V and a normalization time T.

Further, for photons scattered by an equilibrium plasma of chemical potential μ and temperature β^{-1}, there is no interest in knowing the initial and final states of the scattering plasma electron. The scattering rate must therefore be summed over all final states $|f\rangle$ and a Gibbs average taken over initial states $|i\rangle$. In this way we obtain

$$|M|^2 = \frac{e^4VT(2\pi)^4}{4V^2\omega_1\omega_2}\sum_{ifll'}\exp\{\beta(\Omega+\mu N_i-E_i)\}\langle i|\bar{\psi}_\beta|l\rangle\langle l|\psi_\alpha|f\rangle \times\langle f|\bar{\psi}_\sigma|l'\rangle\langle l'|\psi_\tau|i\rangle\tilde{\Gamma}_{\beta\alpha}\Gamma_{\sigma\tau}\delta^4(k_2-k_1+p_f-p_i) \tag{5}$$

with $\Gamma = {}_1\Gamma+{}_2\Gamma$.

It is known (Alekseev 1961) that the expression

$$(2\pi)^4\sum_{ifll'}\exp\{\beta(\Omega+\mu N_i-E_i)\}\langle i|\bar{\psi}|l\rangle\langle l|\psi|f\rangle\langle f|\bar{\psi}|l'\rangle\langle l'|\psi|i\rangle\delta^4(k_2-k_1+p_f-p_i)$$
$$= -\frac{2\,\mathrm{Im}\,K(k_1-k_2)}{1-\exp\{-\beta(\omega_1-\omega_2)\}} \tag{6}$$

where K is the Fourier transform of the retarded two-particle Green function with variables set equal in pairs. Putting $k_1-k_2 = (\boldsymbol{q}, \omega)$ it follows that K is analytic in the upper-half ω plane and is therefore obtained from the equivalent thermodynamic Green function $\mathscr{K}(\boldsymbol{q}, \omega_n)$ for real ω by

$$K(\boldsymbol{q}, \omega) = \mathscr{K}(\boldsymbol{q}, -\mathrm{i}\omega+\delta) \qquad \delta\to 0_+.$$

Since the photons involved in (3) and (4) are real, the presence of the Γ's in (5) does not affect this argument applied to $|M|^2$. This allows (5) to be evaluated by making the substitutions $\kappa_1 = -i\omega_1+\delta_1$, $\kappa_2 = -i\omega_2+\delta_2$ in the expression

$$\frac{2VT}{1-\exp(-\beta\omega)}\frac{1}{\beta}\operatorname{Im} F \tag{7}$$

where F corresponds to the sums over the four possible arrangements of external photons of diagrams of the form in figure 3.

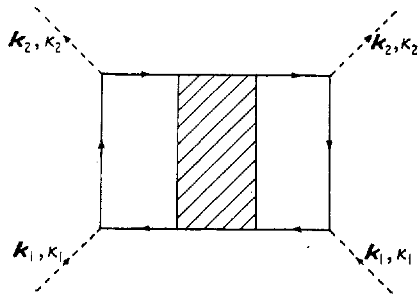

Figure 3. Form of diagrams of F.

In these diagrams the external free-photon lines are assigned expressions $(2V\omega_i)^{-1/2}\epsilon^{(n)}$, just as in quantum electrodynamics, while the remainder is evaluated according to the usual rules for thermodynamic Green functions (Alekseev 1961). To lowest order in e^2 these diagrams are those of figure 4 plus two exchange diagrams which give contributions equal to those of figure 4. The next-order diagrams are obtained by inserting photon lines

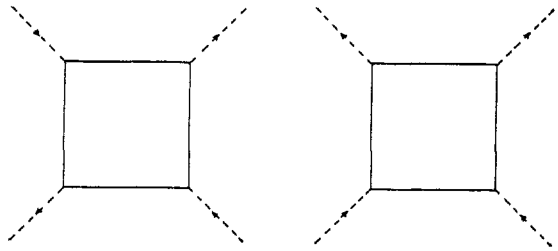

Figure 4. Lowest-order diagrams.

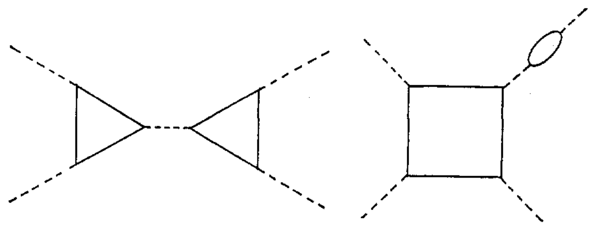

Figure 5. Higher-order diagrams.

and polarization bubbles into those of figure 4. If $\alpha = |qL_D|^{-1}$, with $L_D{}^{-1} = (\beta ne^2)^{1/2}$ the reciprocal Debye length and n the density, is considered to be much smaller than unity, it is possible to neglect diagrams coupling with the ions or other electrons. The remaining higher diagrams give contributions smaller than that of figure 4 by factors of e^2 or $e^4\beta^3 n$ which we take to be negligible, the only interesting cases being those of the types shown in figure 5. The first diagram of figure 5 might be expected to give a large, even dominant, contribution at $k_1 \simeq k_2$, but in fact such diagrams cancel out to exactly zero by Furry's theorem.

The presence of polarization bubbles in the diagrams of the second type introduces a dielectric factor

$$\mathscr{E}^{-1/2}(k) = \{1 - k^{-2}\Pi_{\mu\nu}(k)\}^{-1/2}$$

into the free-photon factor, where $\Pi_{\mu\nu}$ is the usual polarization operator (Alekseev 1961).

For convenience later we wish to maintain a normalization of one particle in a volume V so that the free-photon factor used above must be adjusted for 'modified photons' to $(2V\omega_i)^{-1/2}\mathscr{E}^{1/2}(k_i)\epsilon^{(n)}$. This cancels out the effect of $\mathscr{E}$ on (7), leaving it to be accounted for in the phase-space factors, and is analogous to the practice in classical electromagnetism of replacing e and c by $e\mathscr{E}^{-1/2}$ and $c\mathscr{E}^{-1/2}$ to take account of the dielectric medium (Bekefi 1966). In the random phase approximation and within the imaginary part in (7), $\mathscr{E}$ may be approximated by

$$\mathscr{E} = 1 - \mathrm{Re}\{k^{-2}\Pi_{\nu\nu}(k)\}$$

Π now being the zero-order approximation. This can be evaluated as by Rae (1968) with $k^2 = -\mathrm{i}\delta$, $\delta \to 0$, to correspond to real photons. The non-relativistic case gives

$$\Pi_{\nu\nu}(k_1) = -\frac{2e^2n}{m}\left(1 + \frac{\mathrm{i}\delta}{2\omega_1{}^2}\right)$$

leading to the usual plasma dielectric factor $\mathscr{E} = 1 - \omega_p{}^2/\omega_1{}^2$, ω_p being the plasma frequency $(e^2n/m)^{1/2}$.

From (7) we have for the transition rate per unit volume

$$R = \frac{|M|^2}{VT} = 4\beta^{-1}(1 - \mathrm{e}^{-\beta\omega})^{-1}\,\mathrm{Im}\,F_0 \tag{8}$$

where F_0 corresponds to the sum of the two diagrams in figure 4

$$F_0 = -\frac{e^4}{8V^2\omega_1\omega_2}\frac{1}{(2\pi)^3}\int \mathrm{d}\boldsymbol{p}\sum_{p_4}\mathrm{Tr}\{\Gamma G^0(p)\tilde{\Gamma}G^0(p+q)\} \tag{9}$$

with G^0 the free-electron Green function

$$G^0(p) = (\mathrm{i}\not{p} + m)^{-1}, \qquad p_4 = (2n+1)\pi\beta^{-1} + \mathrm{i}\mu.$$

3. Calculation of the scattering rate

The expression within the summation in (9) can be written

$$\mathrm{Tr}([\not{\epsilon}^{(n)}\{\mathrm{i}(\not{p}+\not{k}_1)+m\}^{-1}\not{\epsilon}^{(m)} + \not{\epsilon}^{(m)}\{\mathrm{i}(\not{p}-\not{k}_2)+m\}^{-1}\not{\epsilon}^{(n)}](\mathrm{i}\not{p}+m)^{-1}[\not{\epsilon}^{(n)}\{\mathrm{i}(\not{p}+\not{k}_1)+m\}^{-1}\not{\epsilon}^{(m)}$$
$$+ \not{\epsilon}^{(m)}\{\mathrm{i}(\not{p}-\not{k}_2)+m\}^{-1}\not{\epsilon}^{(n)}]\{\mathrm{i}(\not{p}+\not{q})+m\}^{-1}).$$

This still contains a complete specification of the polarizations of initial and final photons. We average over these by operating with $\frac{1}{2}\Sigma^2_{m,n=1}$. A well-known argument of Feynman (1949) allows the above to be written in terms of a repeated suffix summation over m, n now ranging from 1 to 4:

$$\tfrac{1}{2}\mathrm{Tr}([\gamma_n\{\mathrm{i}(\not{p}+\not{k}_1)+m\}^{-1}\gamma_m + \gamma_m\{\mathrm{i}(\not{p}-\not{k}_2)+m\}^{-1}\gamma_n](\mathrm{i}\not{p}+m)^{-1}[\gamma_m\{\mathrm{i}(\not{p}+\not{k}_1)+m\}^{-1}\gamma_n$$
$$+ \gamma_n\{\mathrm{i}(\not{p}-\not{k}_2)+m\}^{-1}\gamma_m]\{\mathrm{i}(\not{p}+\not{q})+m\}^{-1}).$$

This may be evaluated in terms of the two traces

$$\begin{aligned} X &= \mathrm{Tr}\{\gamma_n(\not{p}+\not{k}_1+\mathrm{i}m)\gamma_m(\not{p}+\mathrm{i}m)\gamma_m(\not{p}+\not{k}_1+\mathrm{i}m)\gamma_n(\not{p}+\not{q}+\mathrm{i}m)\} \\ &= 16[2\{p(p+k_1)\}\{(p+k_1)(p+q)\} - \{p(p+q)\}\{(p+k_1)^2\}] \\ &\quad - 16m^2(3p^2 + 3pq - 4k_1{}^2 - 4k_1q) + 64m^4 \\ Y &= \mathrm{Tr}\{\gamma_n(\not{p}+\not{k}_1+\mathrm{i}m)\gamma_m(\not{p}+\mathrm{i}m)\gamma_n(\not{p}-\not{k}_2+\mathrm{i}m)\gamma_m(\not{p}+\not{q}+\mathrm{i}m)\} \\ &= -32\{p(p+q)(p+k_1)(p-k_2)\} - 16m^2(6p^2+6pq+q^2-k_1k_2) - 32m^4 \end{aligned}$$

and two other functions

$$D = \{(p+k_1)^2+m^2\}(p^2+m^2)\{(p+k_1)^2+m^2\}\{(p+q)^2+m^2\}$$
$$E = \{(p+k_1)^2+m^2\}(p^2+m^2)\{(p-k_2)^2+m^2\}\{(p+q)^2+m^2\}.$$

The function F_0 of equation (9) now becomes

$$F_0 = -\frac{e^4}{8V^2\omega_1\omega_2}\frac{1}{(2\pi)^3}\int \mathrm{d}\boldsymbol{p}\sum_{p_4}\left(\frac{X}{D}+\frac{Y}{E}\right). \tag{10}$$

We consider first the term Y/E and split it into partial fractions in the variable p_4, with no repeated factors. If the resulting terms are combined in pairs, each with its complex conjugate, the sum over p_4 can be performed using the standard result

$$\frac{2\epsilon}{\beta}\sum_{p_4}(\epsilon^2+p_4{}^2)^{-1} = 1-n^-(p)-n^+(p) = 1-n(p)$$

where $\epsilon = (\boldsymbol{p}^2+m^2)^{1/2}$ and $n^{\pm}(p) = \{1+\exp\beta(\epsilon\pm\mu)\}^{-1}$ is the electron–positron distribution function. The 1 in this formula leads to various divergences which are exactly those occurring in vacuum electrodynamics and are thus removed by renormalizing in the usual way. The resulting expression is simplified by putting $k_1{}^2 = k_2{}^2 = 0$ and, after a large amount of elementary algebra, can be expressed in the form

$$\sum_{p_4}\frac{Y}{E} = \frac{\beta}{\epsilon}\frac{-4n(\boldsymbol{p})}{\{(\boldsymbol{p}\,.\,\boldsymbol{k}_1)^2-\epsilon^2\omega_1{}^2\}\{(\boldsymbol{p}\,.\,\boldsymbol{k}_2)^2-\epsilon^2\omega_2{}^2\}}\left\{\frac{\Theta(k_1,k_2,\boldsymbol{p})}{(2\boldsymbol{p}\,.\,\boldsymbol{q}+q^2)^2-4\epsilon^2\omega^2}+\frac{\Theta(k_1,-k_2,\boldsymbol{p})}{(2\boldsymbol{p}\,.\,\boldsymbol{r}+r^2)^2-4\epsilon^2 r_4{}^2}\right\} \tag{11}$$

where $q = k_1-k_2$, $r = k_1+k_2$. For equation (8) we require only the imaginary part of this, which arises from poles of the integrand in (10). The second term of (11) has no poles for real $\boldsymbol{p}$ and thus contributes nothing. The function Θ is given by

$$\begin{aligned}\Theta(k_1,k_2,\boldsymbol{p}) = {} & \{2(\boldsymbol{p}\,.\,\boldsymbol{q})^2+2m^2\boldsymbol{p}\,.\,\boldsymbol{q}-2m^4+q^2\boldsymbol{p}\,.\,\boldsymbol{q}+\tfrac{1}{2}m^2q^2\}\{(2\boldsymbol{p}\,.\,\boldsymbol{q}+q^2)\{(\boldsymbol{p}\,.\,\boldsymbol{k}_1\;\boldsymbol{p}\,.\,\boldsymbol{k}_2)\\ & +\epsilon^2\omega_1\omega_2\}+2\epsilon^2\omega(\omega_1\boldsymbol{p}\,.\,\boldsymbol{k}_2+\omega_2\boldsymbol{p}\,.\,\boldsymbol{k}_1)\}\\ & -\epsilon^2\omega(m^2+2\boldsymbol{p}\,.\,\boldsymbol{q}+\tfrac{1}{2}q^2)\{2\omega(\boldsymbol{p}\,.\,\boldsymbol{k}_1\;\boldsymbol{p}\,.\,\boldsymbol{k}_2+\epsilon^2\omega_1\omega_2)\\ & +(2\boldsymbol{p}\,.\,\boldsymbol{q}+q^2)(\omega_1\boldsymbol{p}\,.\,\boldsymbol{k}_2+\omega_2\boldsymbol{p}\,.\,\boldsymbol{k}_1)\}.\end{aligned}$$

We now return to (10) and look for a similar expression for X/D. In this case there is a repeated factor which complicates the evaluation of the partial fractions. However, the terms from this have no poles for real $\boldsymbol{p}$ and can be discarded just as above. The terms from the non-repeated factor can be evaluated in the same way as (11) to give

$$\sum_{p_4}\frac{X}{D} = \frac{\beta}{\epsilon}\frac{-2n(\boldsymbol{p})}{\{(2\boldsymbol{p}\,.\,\boldsymbol{q}+q^2)^2-4\epsilon^2\omega^2\}}\left[\frac{\Phi(k_1,k_2,\boldsymbol{p})}{\{(\boldsymbol{p}\,.\,\boldsymbol{k}_1)^2-\epsilon^2\omega_1{}^2\}^2}+\frac{\Phi(-k_2,-k_1,\boldsymbol{p})}{\{(\boldsymbol{p}\,.\,\boldsymbol{k}_2)^2-\epsilon^2\omega_2{}^2\}^2}\right] \tag{13}$$

where

$$\begin{aligned}\Phi(k_1,k_2,\boldsymbol{p}) = {} & \{(\boldsymbol{p}\,.\,\boldsymbol{k}_1)^2q^2+2(\boldsymbol{p}\,.\,\boldsymbol{k}_1)^2\boldsymbol{p}\,.\,\boldsymbol{q}+\epsilon^2\omega_1{}^2q^2+2\epsilon^2\omega_1{}^2\boldsymbol{p}\,.\,\boldsymbol{q}+4\epsilon^2\boldsymbol{p}\,.\,\boldsymbol{k}_1\omega_1\omega\}\\ & \times\{2m^4-2m^2\boldsymbol{p}\,.\,\boldsymbol{k}_1+2m^2\boldsymbol{p}\,.\,\boldsymbol{q}+2\epsilon^2\omega_1{}^2+m^2q^2+\boldsymbol{p}\,.\,\boldsymbol{k}_1q^2\}\\ & -2\epsilon^2\{\omega_1q^2\boldsymbol{p}\,.\,\boldsymbol{k}_1+2\omega_1\,\boldsymbol{p}\,.\,\boldsymbol{k}_1\;\boldsymbol{p}\,.\,\boldsymbol{q}+\omega(\boldsymbol{p}\,.\,\boldsymbol{k}_1)^2+\epsilon^2\omega_1{}^2\omega\}(\omega_1q^2+4\omega_1\boldsymbol{p}\,.\,\boldsymbol{k}_1\\ & +2m^2\omega-2m^2\omega_1)\end{aligned} \tag{14}$$

within the ranges where $e^4\beta^3n$ and $\beta ne^2\omega_1{}^{-2}$ are small and, considering scattering away from the zero angle, we thus obtain from equations (8) and (10) to (14) a formula for the scattering rate

$$R = \frac{e^4}{(2\pi)^3 V^2} \frac{1}{\omega_1\omega_2\{1-\exp(-\beta\omega)\}} \mathrm{Im} \int \mathrm{d}\boldsymbol{p} \frac{n(p)}{\epsilon} \frac{1}{(2\boldsymbol{p}\cdot\boldsymbol{q}+q^2)^2 - 4\epsilon^2\omega^2}$$
$$\times \left[\frac{\Phi(k_1, k_2, \boldsymbol{p})}{\{(\boldsymbol{p}\cdot\boldsymbol{k}_1)^2 - \epsilon^2\omega_1^2\}^2} + \frac{\Phi(-k_2, -k_1, \boldsymbol{p})}{\{(\boldsymbol{p}\cdot\boldsymbol{k}_2)^2 - \epsilon^2\omega_2^2\}^2} + \frac{2\Theta(k_1, k_2, \boldsymbol{p})}{\{(\boldsymbol{p}\cdot\boldsymbol{k}_1)^2 - \epsilon^2\omega_1^2\}\{(\boldsymbol{p}\cdot\boldsymbol{k}_2)^2 - \epsilon^2\omega_2^2\}}\right]. \tag{15}$$

The intensity of the incident beam is

$$I = \frac{1}{V}\frac{\partial\omega_1}{\partial\boldsymbol{k}_1}$$

and the density of final states is

$$\frac{V}{(2\pi)^3}\mathrm{d}\boldsymbol{k}_2$$

where $\boldsymbol{k}_i^2 = \mathscr{E}(\boldsymbol{k}_i, \omega_i)\omega_i^2$. These allow the differential scattering cross section to be computed from (15).

4. The non-relativistic case

Expression (15) is too complicated to handle generally but may serve as a starting point for specializing to different realms of physical interest. We show here how, in the simplest case (the non-relativistic case) the formulae reduce to known results. For this we have $\beta m \gg 1$, $\omega_1 \ll m$ and $n(p)$ a Maxwellian distribution.

With a change of variable from $\boldsymbol{p}$ to $\boldsymbol{p}-\frac{1}{2}\boldsymbol{q}$ the relevant approximations to Φ and Θ are, from (12) and (14),

$$\begin{aligned}\Theta &= (-m^2q^2 - 2m^4 + \tfrac{1}{2}m^2q^2)(2m^2\omega_1\omega_2\boldsymbol{p}\cdot\boldsymbol{q}) \\ &= 2m^2\omega_1\omega_2\boldsymbol{p}\cdot\boldsymbol{q}(-2m^4 - \tfrac{1}{2}m^2q^2) \\ \Phi &= (2m^2\omega_1^2\boldsymbol{p}\cdot\boldsymbol{q})(2m^4 + m^2\boldsymbol{q}\cdot\boldsymbol{k}_1 + 2m^2\omega_1^2) - 2m^2\{-\omega_1\boldsymbol{p}\cdot\boldsymbol{q}\;\boldsymbol{q}\cdot\boldsymbol{k}_1\}(-2m^2\omega_2) \\ &= 2m^2\omega_1\boldsymbol{p}\cdot\boldsymbol{q}(2m^4\omega_1 + 2m^2\omega_1^3 + m^2\omega_1\boldsymbol{q}\cdot\boldsymbol{k}_1 - 2m^2\omega_2\boldsymbol{q}\cdot\boldsymbol{k}_1).\end{aligned}$$

Then (15) becomes

$$R = \frac{e^4}{(2\pi)^3V^2}\frac{\{1-\exp(-\beta\omega)\}^{-1}}{2m\omega_1\omega_2}\left(\frac{2m^2\omega^2}{\omega_1^2\omega_2^2} - \frac{2q^2}{\omega_1\omega_2} + 4\right)$$
$$\times \mathrm{Im}\int \mathrm{d}\boldsymbol{p}\frac{\boldsymbol{p}\cdot\boldsymbol{q}\exp[-\beta\{\epsilon(\boldsymbol{p}-\frac{1}{2}\boldsymbol{q})-\mu\}]}{(\boldsymbol{p}\cdot\boldsymbol{q})^2 - m^2\omega^2}. \tag{16}$$

The required imaginary part is

$$\tfrac{1}{2}\mathrm{Im}\int \mathrm{d}\boldsymbol{p}\exp(\mu\beta)\frac{\exp\{-\beta\epsilon(\boldsymbol{p}-\frac{1}{2}\boldsymbol{q})\} - \exp\{-\beta\epsilon(\boldsymbol{p}+\frac{1}{2}\boldsymbol{q})\}}{\boldsymbol{p}\cdot\boldsymbol{q} - m\omega}$$
$$= \frac{\pi^2 m}{\beta q}\exp(\mu\beta)\{1-\exp(-\beta\omega)\}\exp\left\{-\tfrac{1}{2}m\beta\left(\frac{\omega}{q} - \frac{q}{2m}\right)^2\right\}.$$

With this exponential factor the factor within $\left(\;\right)$ in equation (16) has the value $2(1+\cos^2\phi)$, where ϕ is the angle between $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$, so that the scattering rate is

$$\begin{aligned}R &= \frac{e^4}{(2\pi)^3V^2}\frac{1+\cos^2\phi}{m\omega_1\omega_2}\frac{\pi^2 m}{\beta q}\frac{1}{2}\left(\frac{2\pi\beta}{m}\right)^{3/2} n\exp\left\{-\tfrac{1}{2}\beta m\left(\frac{\omega}{q} - \frac{q}{2m}\right)^2\right\} \\ &= \frac{2\pi^2 r_0^2}{V^2}(2\pi\beta m)^{1/2} n\frac{(1+\cos^2\phi)}{\omega_1\omega_2 q}\exp\left\{-\tfrac{1}{2}\beta m\left(\frac{\omega}{q} - \frac{q}{2m}\right)^2\right\}\end{aligned}$$

with $r_0 = e^2/4\pi m$ the classical electron radius.

The dielectric function is that evaluated in § 2 so that the intensity of the incident beam is

$$I = \frac{\{\mathscr{E}(\omega_1)\}^{1/2}}{V} \simeq \frac{1}{V}$$

and the density of final states is

$$\frac{V}{(2\pi)^3}\{\mathscr{E}(\omega_2)\}^{1/2}\omega_2{}^2\,d\omega_2\,d\Omega.$$

This gives for the cross section in plasma units, i.e. momenta in units of $L_D{}^{-1}$ and energies in units of ω_p (Dubois 1964 a),

$$\frac{d\sigma(\boldsymbol{q},\omega)}{d\omega\,d\Omega} = \frac{nr_0{}^2}{(2\pi)^{1/2}}\frac{\{\mathscr{E}(\omega_2)\}^{1/2}}{q}\tfrac{1}{2}(1+\cos^2\phi)\frac{\omega_2}{\omega_1}\exp\left\{-\frac{1}{2}\left(\frac{\omega}{q}-\tfrac{1}{2}\hbar q\right)^2\right\}$$

which agrees with well-known expressions (Akhiezer 1967, Dubois 1964 a) for Doppler-broadened scattering from free electrons.

5. Conclusions

Modern Green function and diagram techniques allow the Compton cross section for scatter in plasmas to be derived from basic assumptions about statistical physics and electromagnetism. Assuming the physical parameters are such that $e^4\beta^3 n$ and $e^2\beta n\omega_1{}^{-2}$ are small and considering scattering away from zero angle, a general expression (15) is obtained for scattering rates for a wide range of temperature and energy. The calculations of cross sections for different physical regions are best done separately, the method being illustrated by deducing the known result in the non-relativistic case.

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